

Q_1 : Find the k -th smallest element between indexes i and j

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Q_1 : Find the k -th smallest element between indexes i and j

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

Q_1 : Find the k -th smallest element between indexes i and j

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

3	4	4	5	6	7	7	8	9	9
---	---	---	---	---	---	---	---	---	---

Q_1 : Find the k -th smallest element between indexes i and j

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

3	4	4	5	6	7	7	8	9	9
---	---	---	---	---	---	---	---	---	---




Q_2 : Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---


Q_2 : Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



Q_2 : Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



Q_2 : Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	2	9	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Q₂: Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	2	9	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

Q₂: Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	2	9	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

2	3	4	4	5	6	7	7	8	9
---	---	---	---	---	---	---	---	---	---

Q₂: Swap the elements at positions i and $i + 1$

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	2	9	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

4th smallest element?

2	3	4	4	5	6	7	7	8	9
---	---	---	---	---	---	---	---	---	---

↑

Input

- ▶ sequence of length $N = 10^6$
- ▶ values in the range $[-10^9, 10^9]$
- ▶ 10^5 (online) queries of either type Q_1 or Q_2

Task

- ▶ report the answer to queries of type Q_1 considering the updates Q_2

Wavelet Trees for Competitive Programming

Robinson Castro, Nico Lehmann, Jorge Pérez, Bernardo Subercaseaux

Universidad de Chile, École Centrale Paris

Wavelet Tree

- ▶ represents sequences over a (potentially big) alphabet Σ
- ▶ basic (tree) structure can be built in $O(N \times \log |\Sigma|)$
- ▶ answer several interesting queries in time $O(\log |\Sigma|)$

Wavelet Tree

- ▶ represents sequences over a (potentially big) alphabet Σ
- ▶ basic (tree) structure can be built in $O(N \times \log |\Sigma|)$
- ▶ answer several interesting queries in time $O(\log |\Sigma|)$
- ▶ can be smoothly composed with more classical structures

3 SPOJ problems solvable with Wavelet Trees

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

3 SPOJ problems solvable with Wavelet Trees

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

The Wavelet Tree

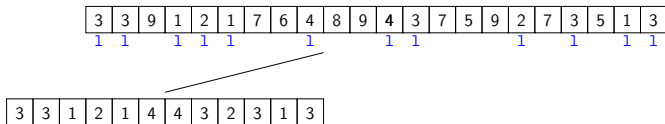
Wavelet Tree construction

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Wavelet Tree construction

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
1	1		1	1	1			1			1	1				1		1		1	1

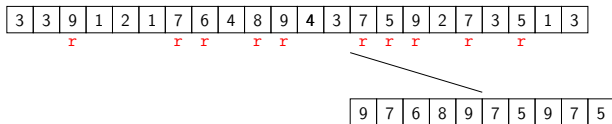
Wavelet Tree construction



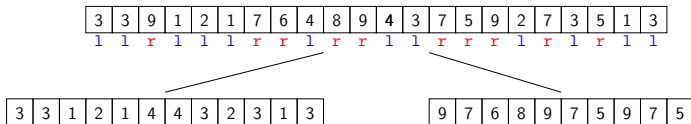
Wavelet Tree construction

3	3	9	1	2	1	7	6	4	8	9	4	3	7	5	9	2	7	3	5	1	3
		r				r	r		r	r		r	r	r		r		r			

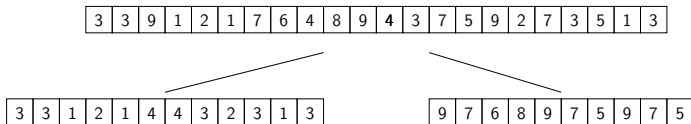
Wavelet Tree construction



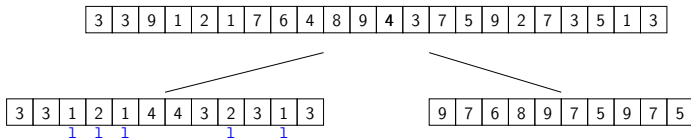
Wavelet Tree construction



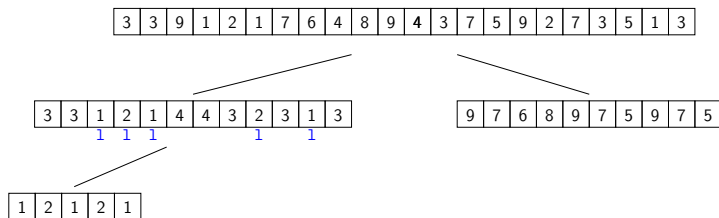
Wavelet Tree construction



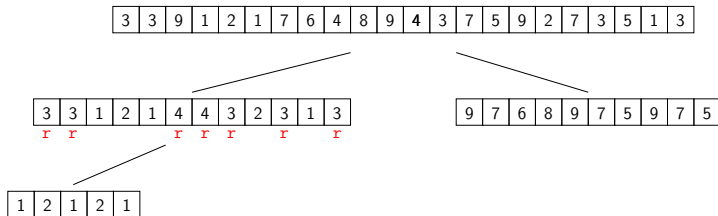
Wavelet Tree construction



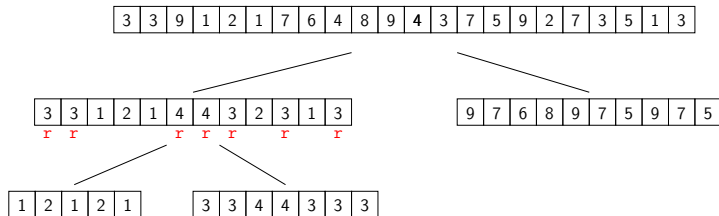
Wavelet Tree construction



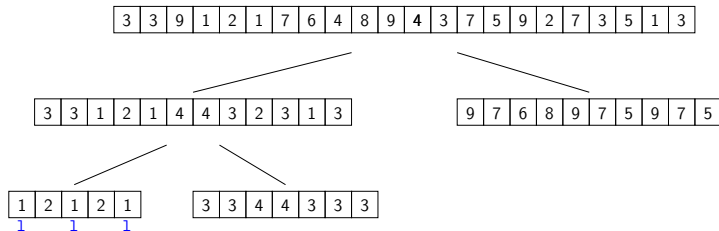
Wavelet Tree construction



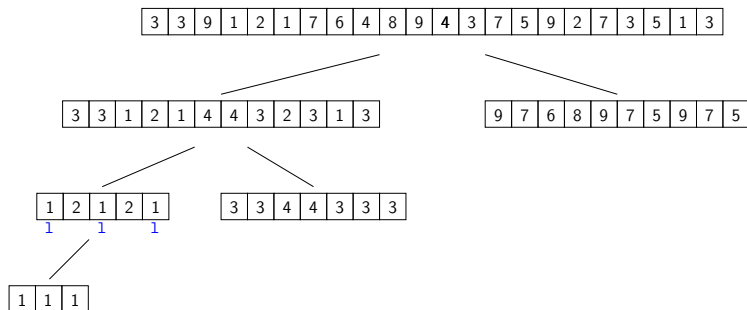
Wavelet Tree construction



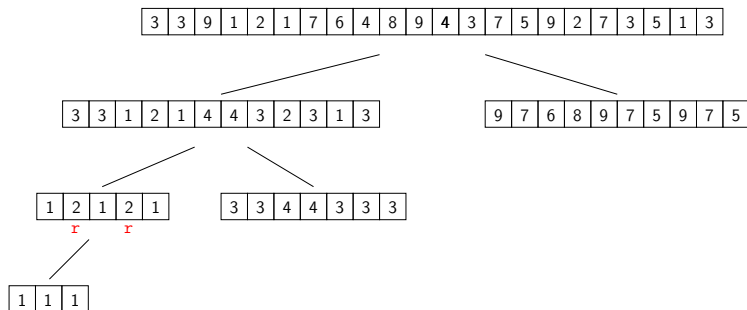
Wavelet Tree construction



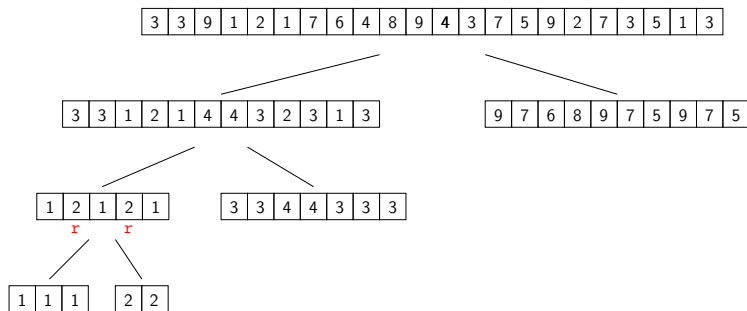
Wavelet Tree construction



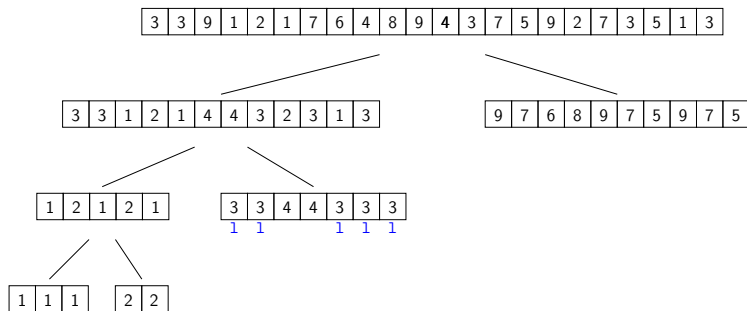
Wavelet Tree construction



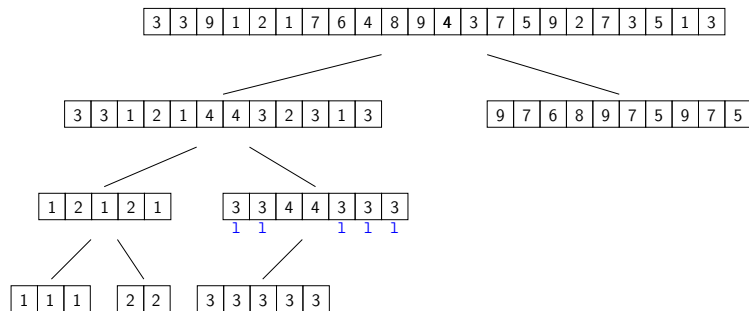
Wavelet Tree construction



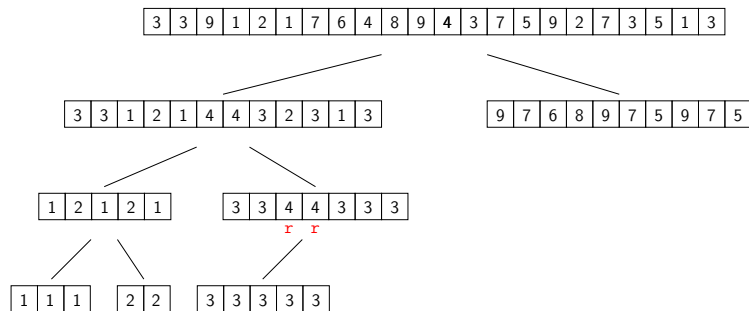
Wavelet Tree construction



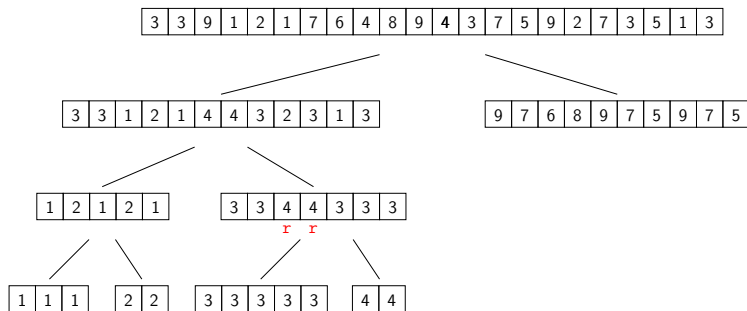
Wavelet Tree construction



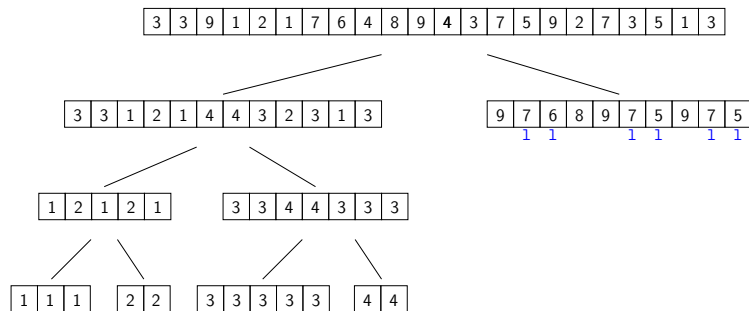
Wavelet Tree construction



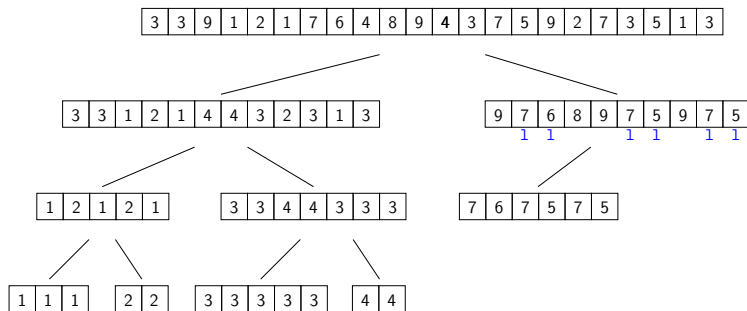
Wavelet Tree construction



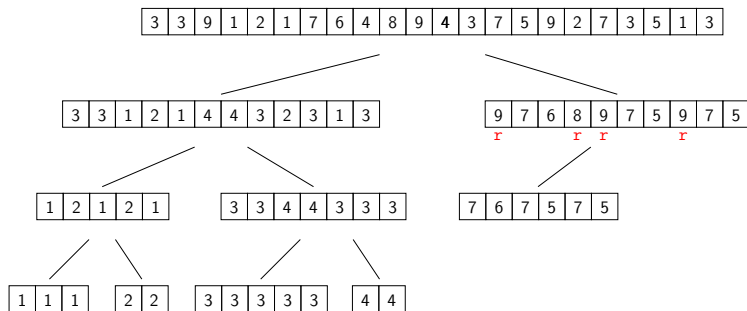
Wavelet Tree construction



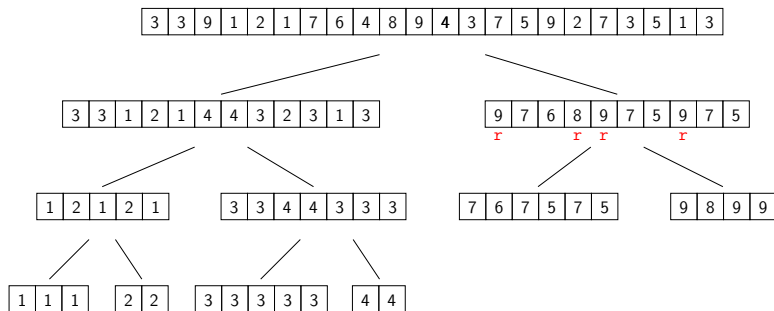
Wavelet Tree construction



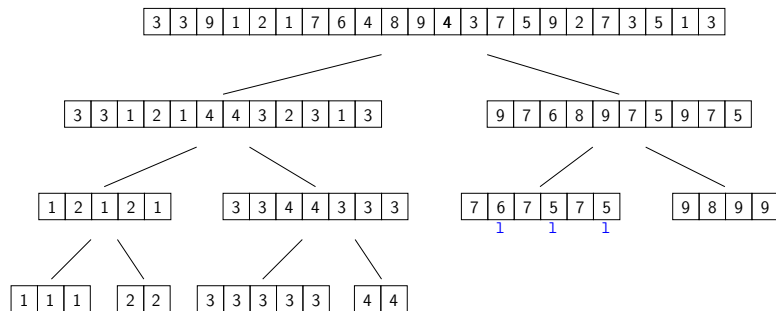
Wavelet Tree construction



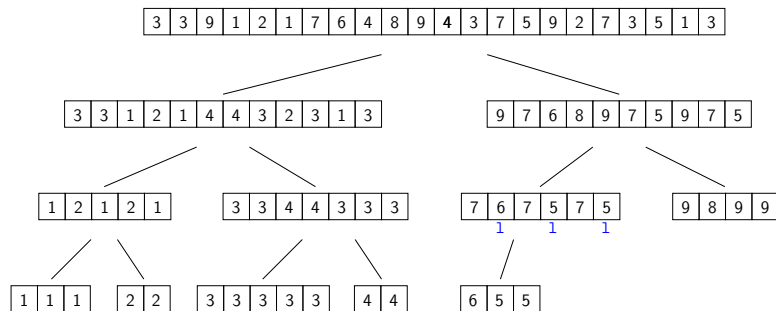
Wavelet Tree construction



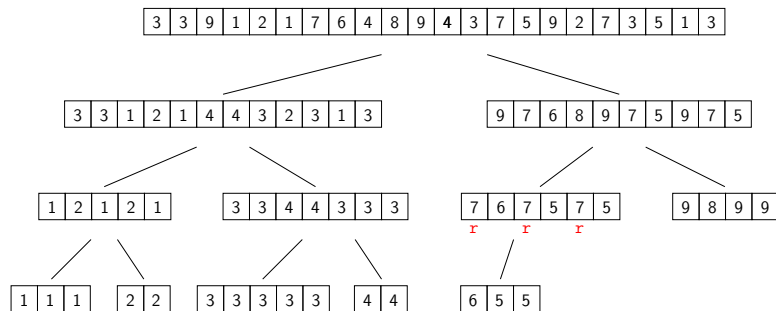
Wavelet Tree construction



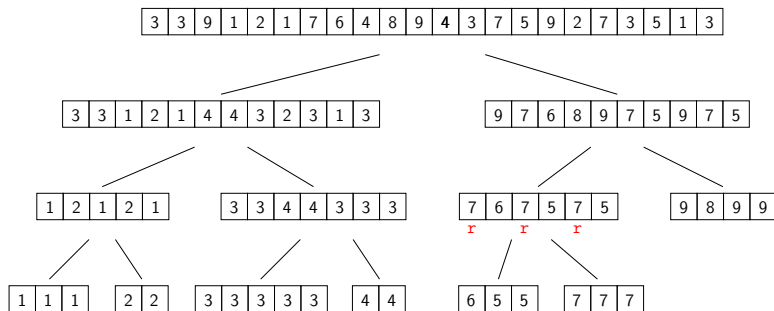
Wavelet Tree construction



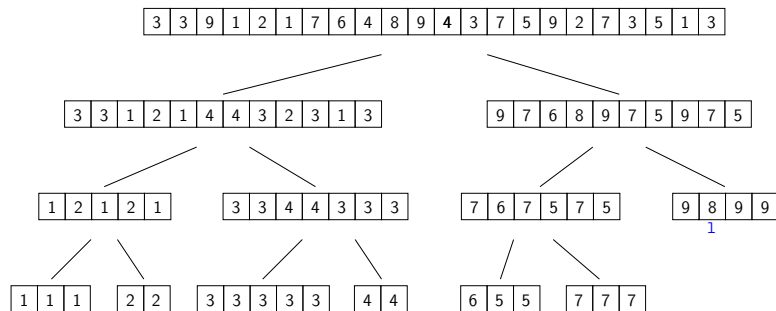
Wavelet Tree construction



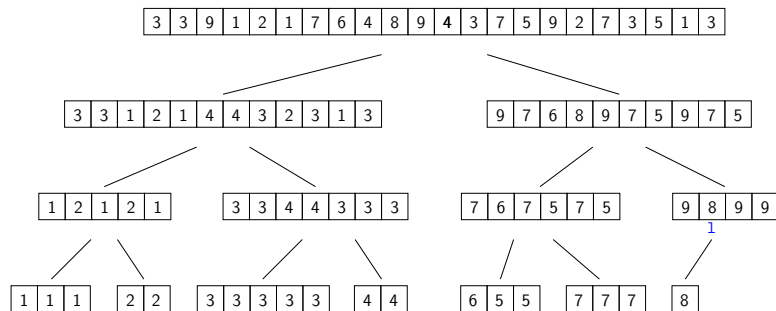
Wavelet Tree construction



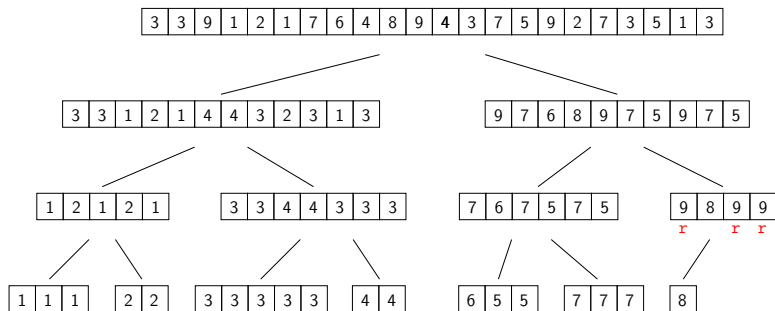
Wavelet Tree construction



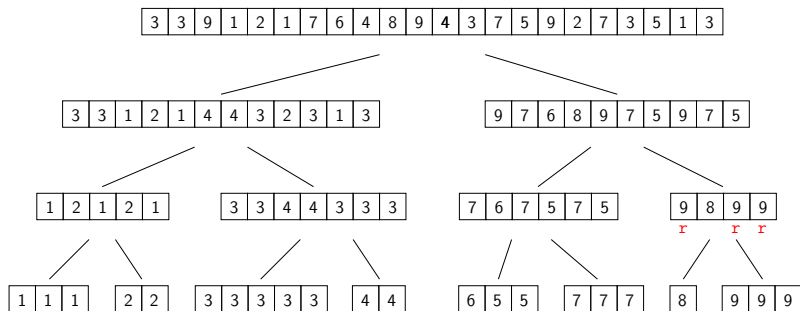
Wavelet Tree construction



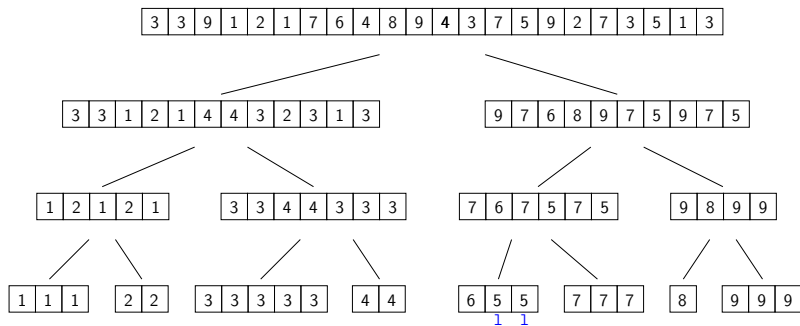
Wavelet Tree construction



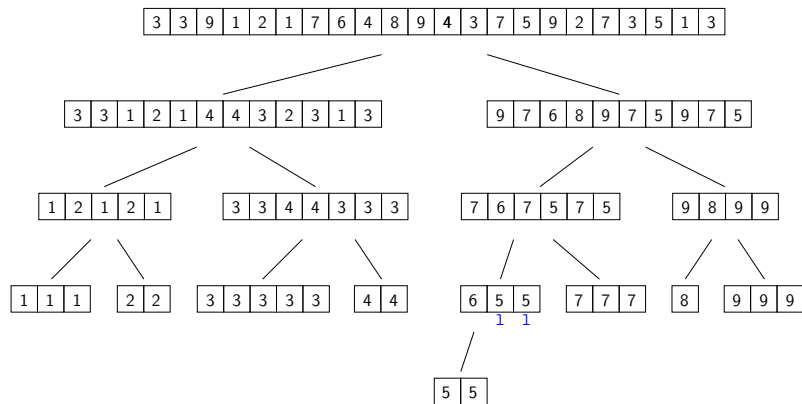
Wavelet Tree construction



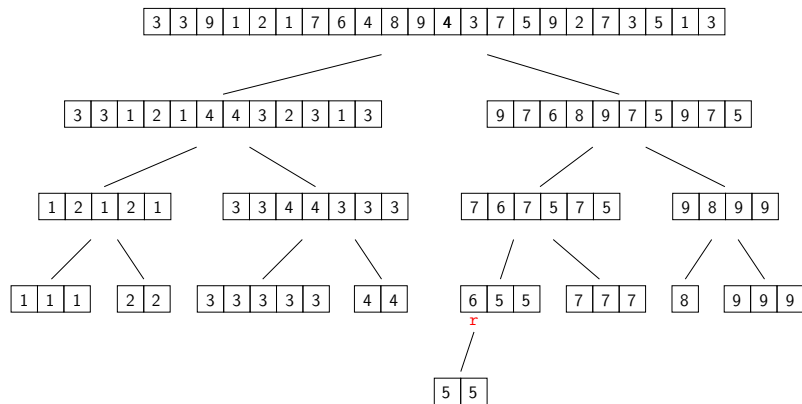
Wavelet Tree construction



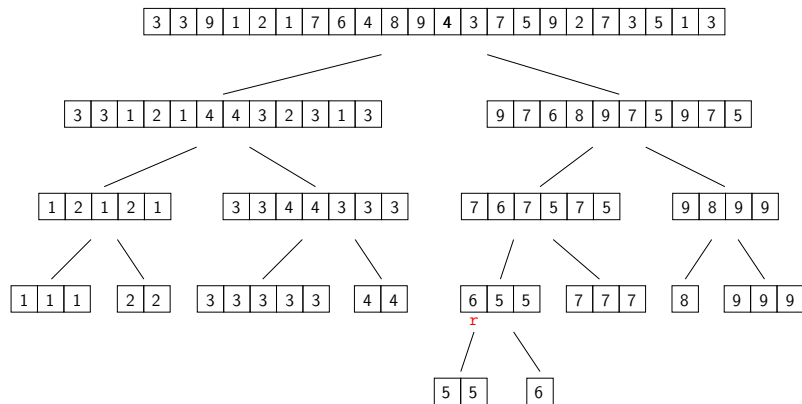
Wavelet Tree construction



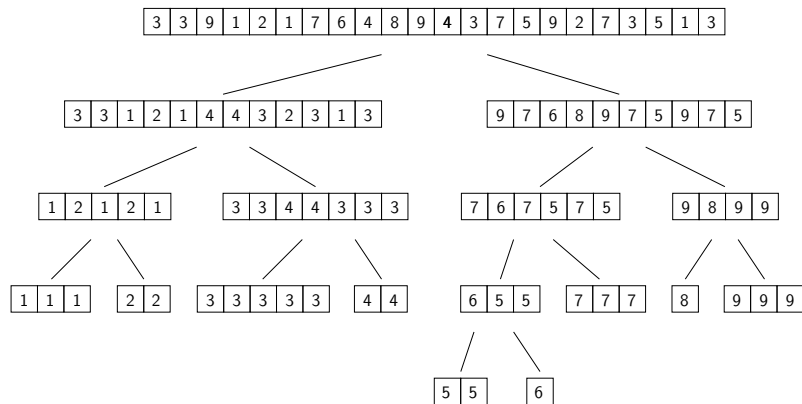
Wavelet Tree construction



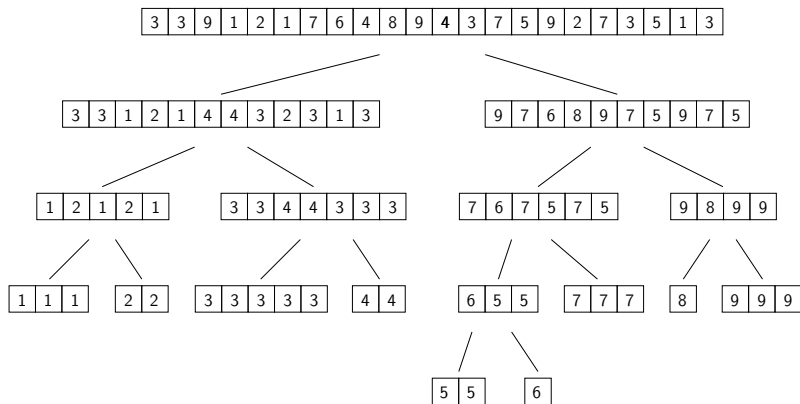
Wavelet Tree construction



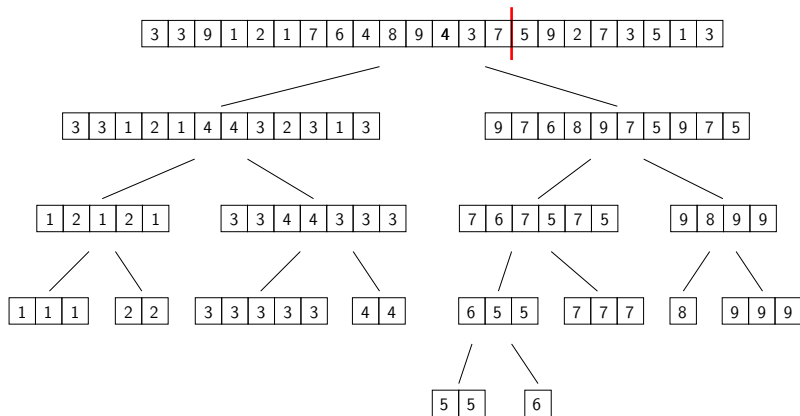
Wavelet Tree construction



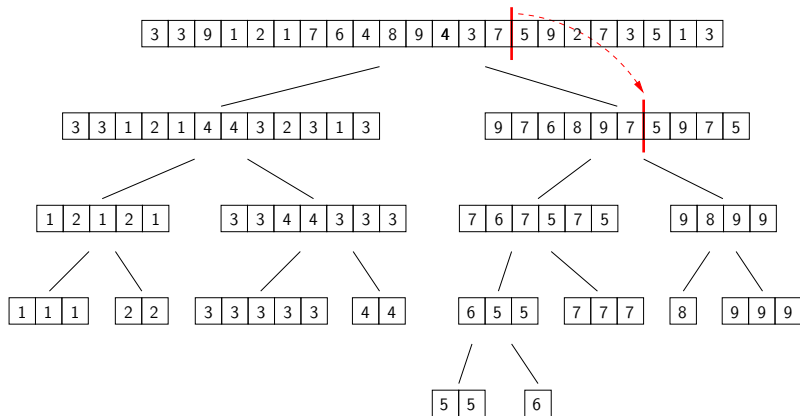
Mapping indexes to the left and to the right



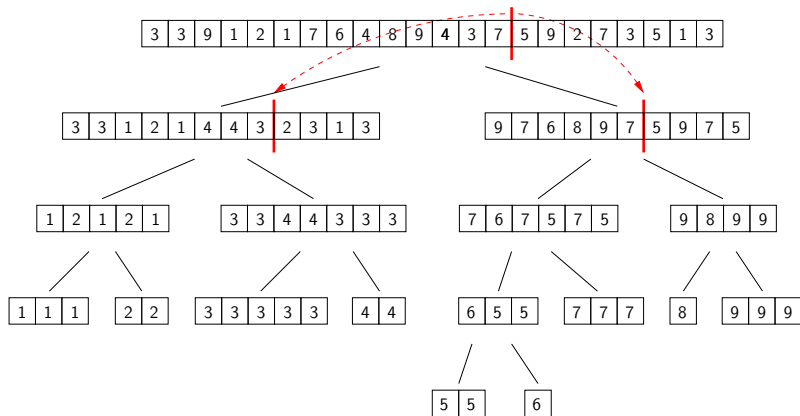
Mapping indexes to the left and to the right



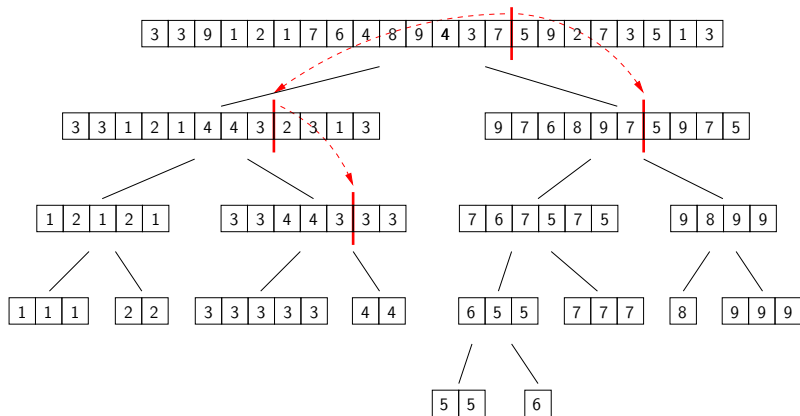
Mapping indexes to the left and to the right



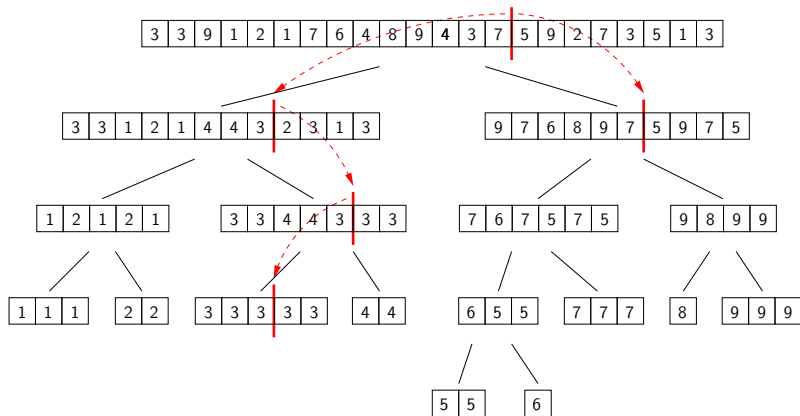
Mapping indexes to the left and to the right



Mapping indexes to the left and to the right

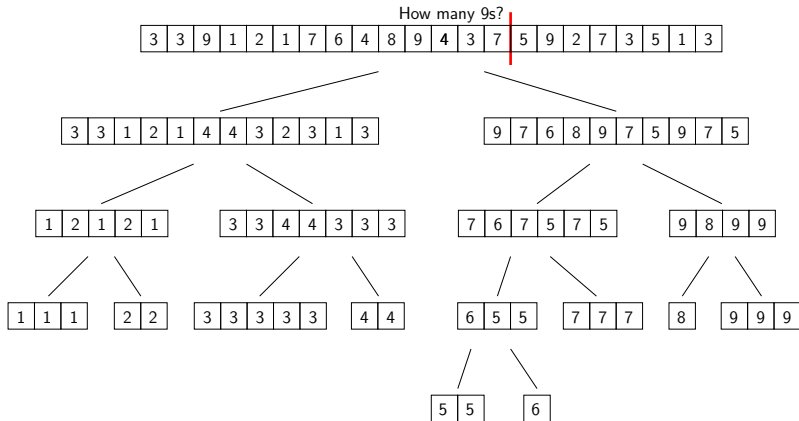


Mapping indexes to the left and to the right

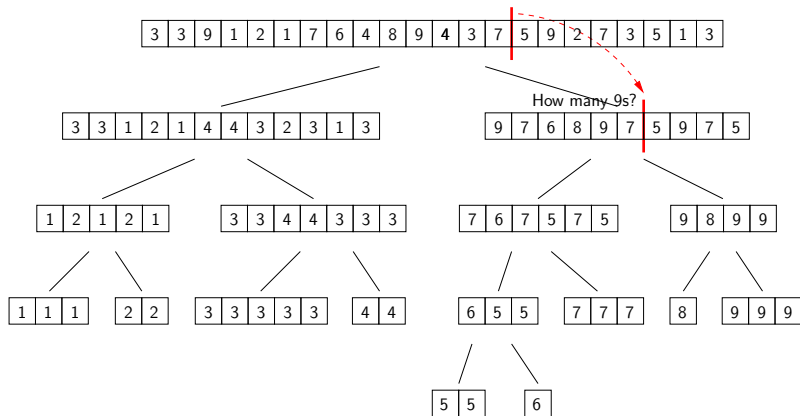


Queries

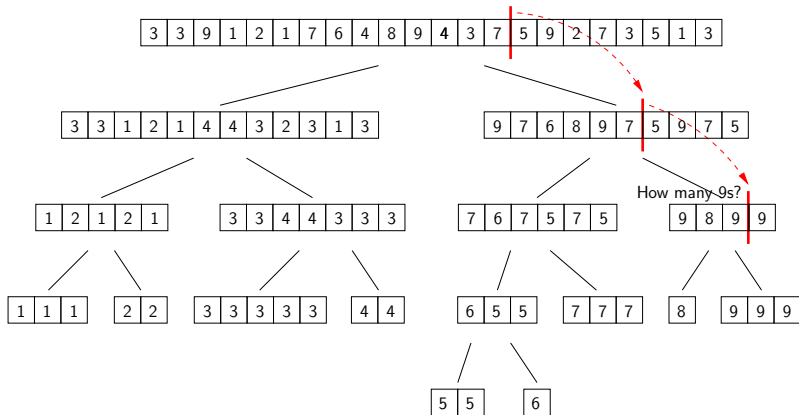
Rank queries



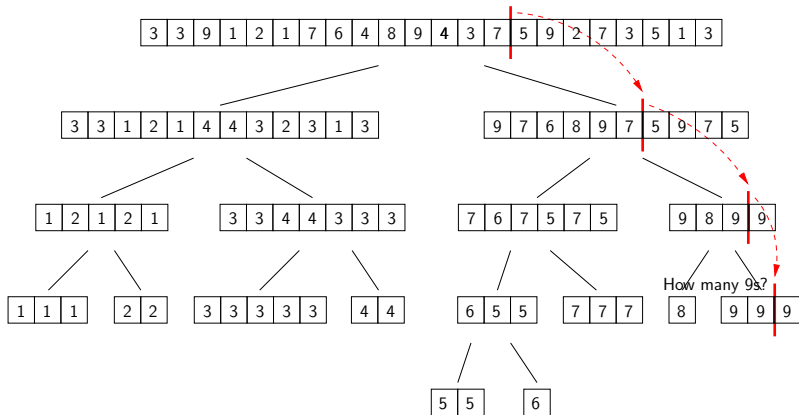
Rank queries



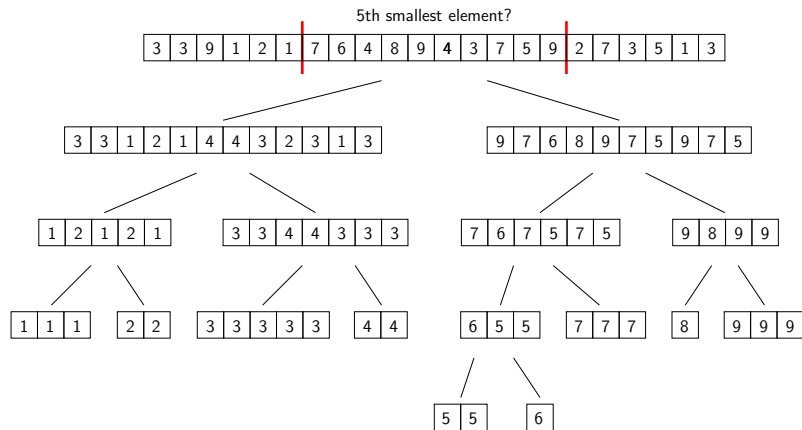
Rank queries



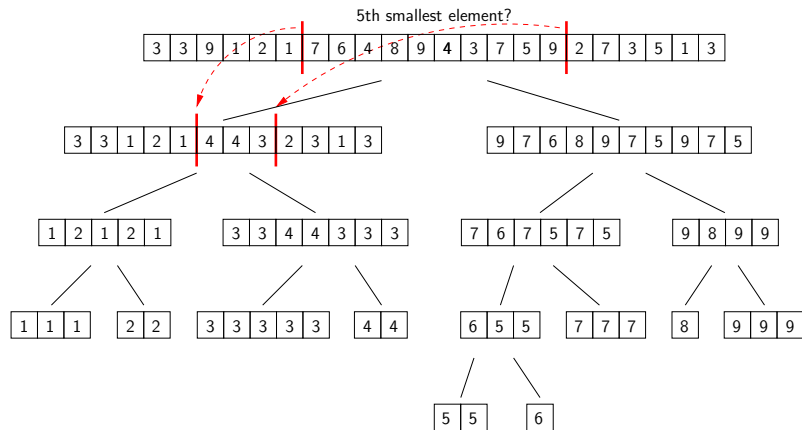
Rank queries



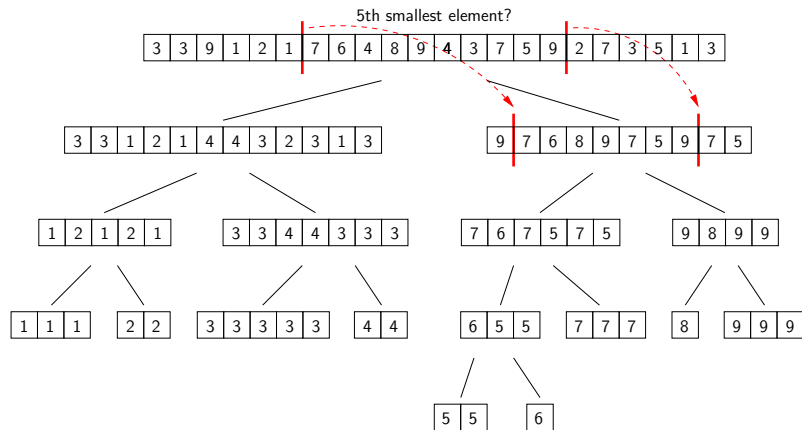
Quantile queries



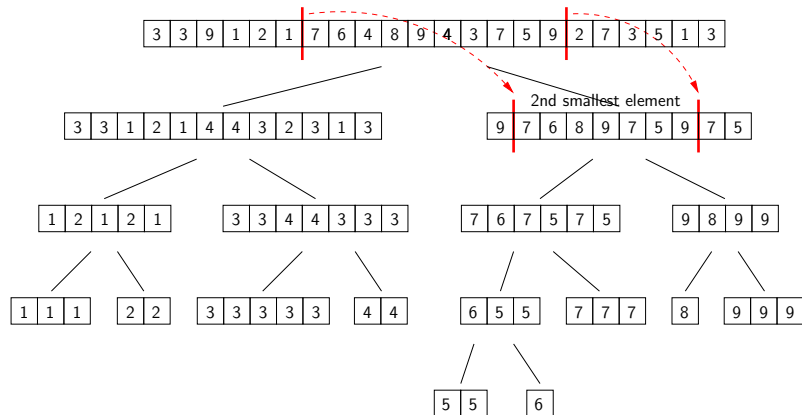
Quantile queries



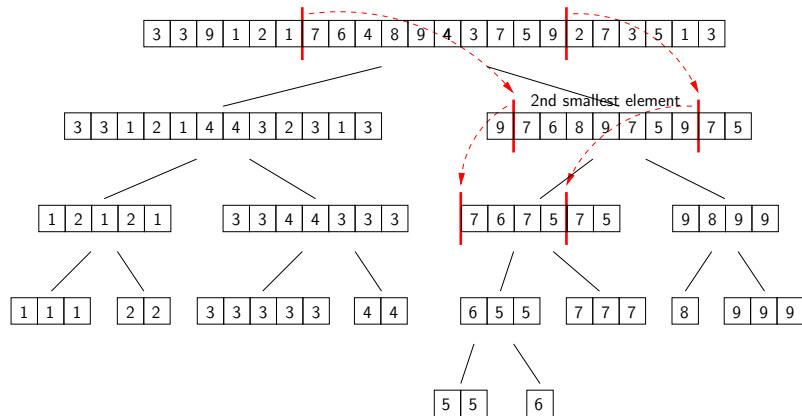
Quantile queries



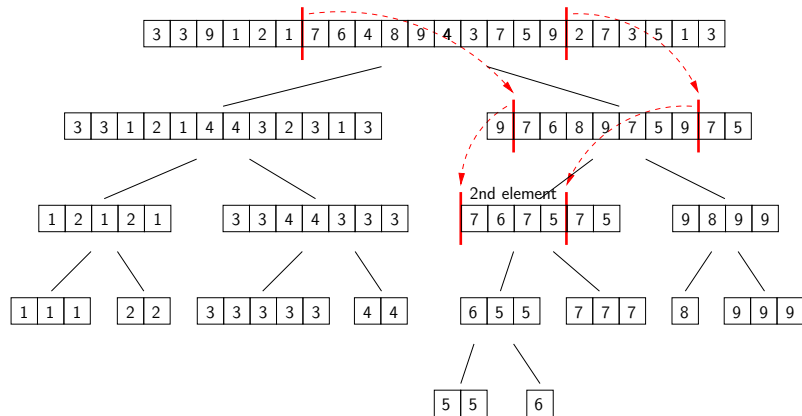
Quantile queries



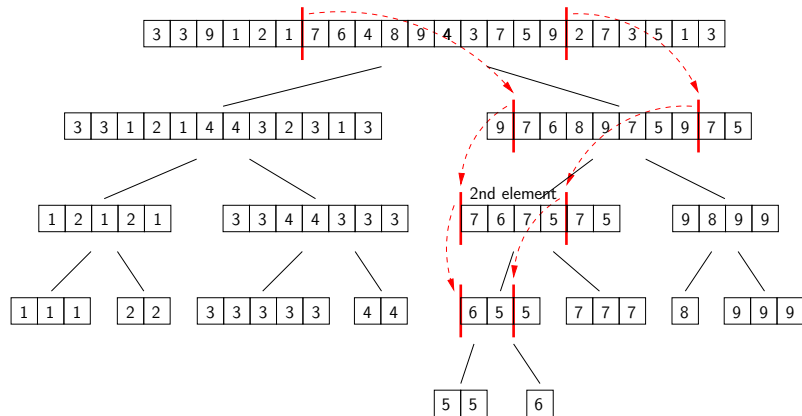
Quantile queries



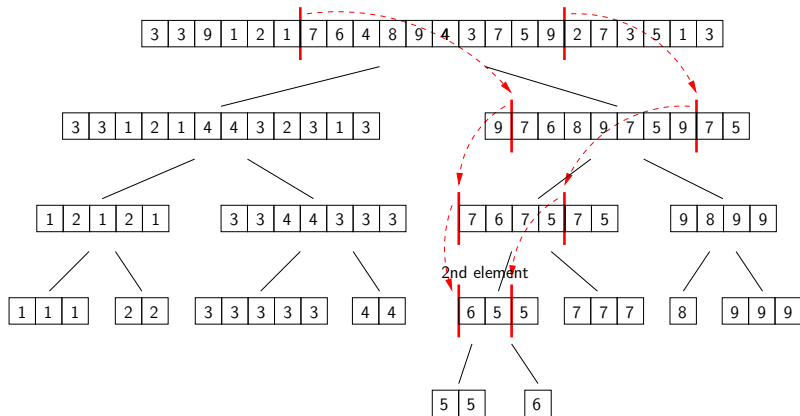
Quantile queries



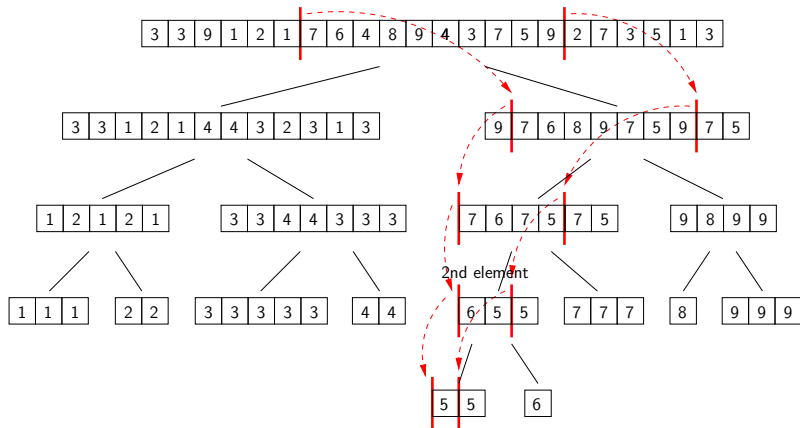
Quantile queries



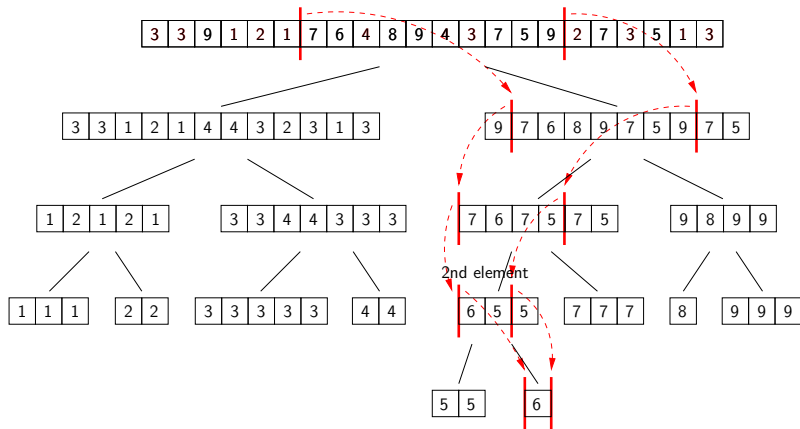
Quantile queries



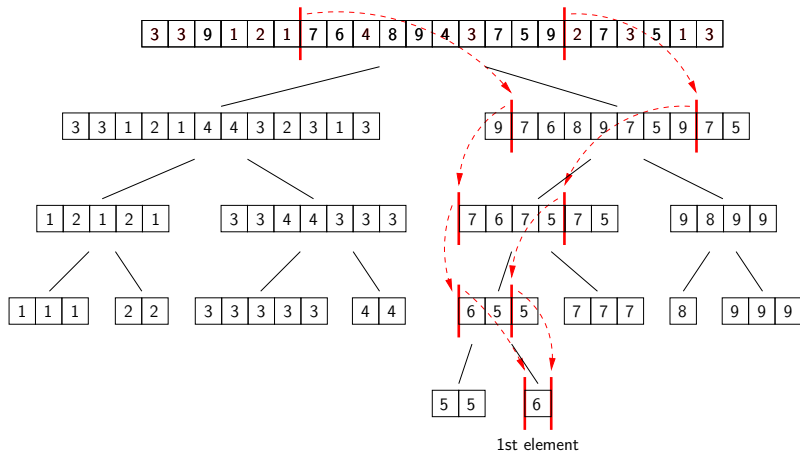
Quantile queries



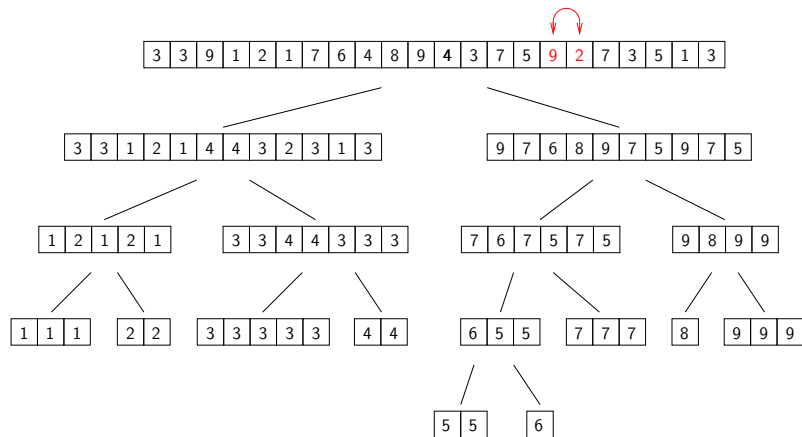
Quantile queries



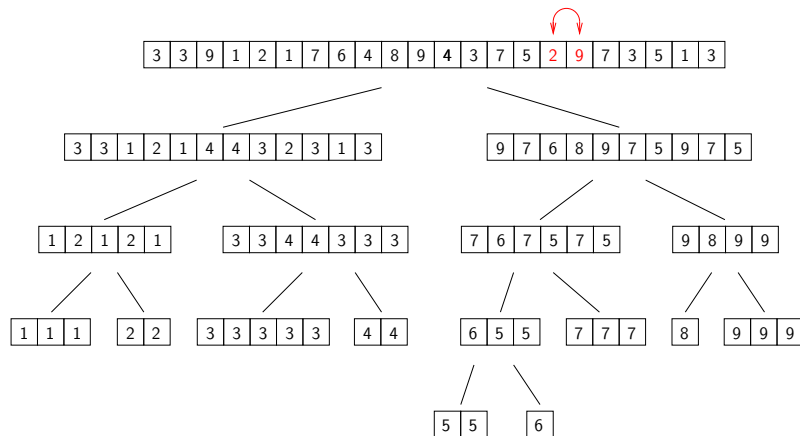
Quantile queries



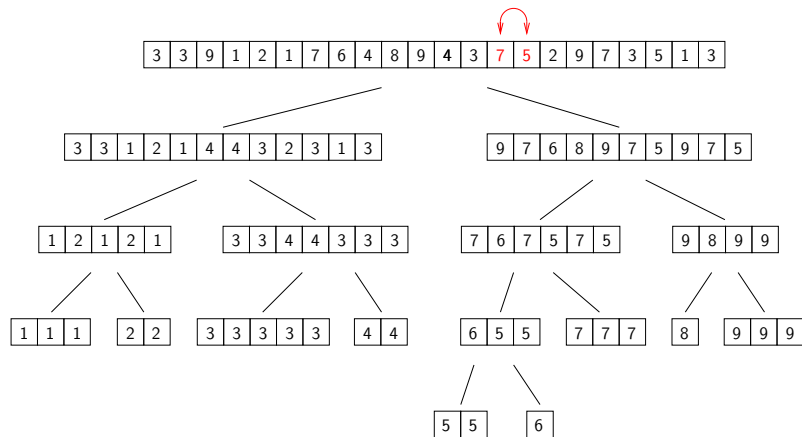
Contiguous Swap



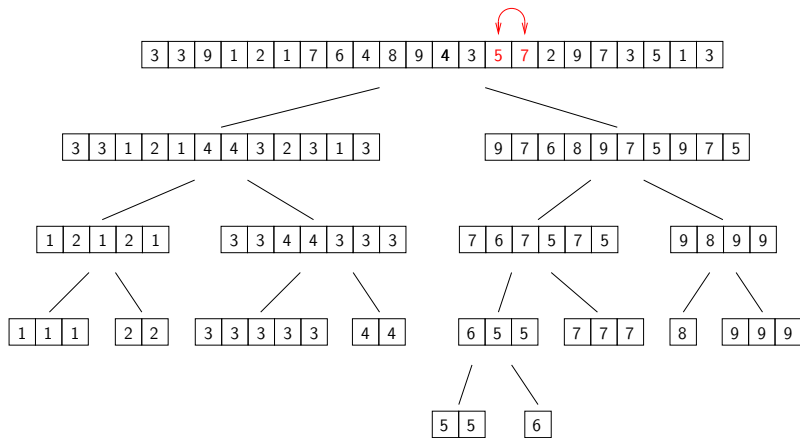
Contiguous Swap



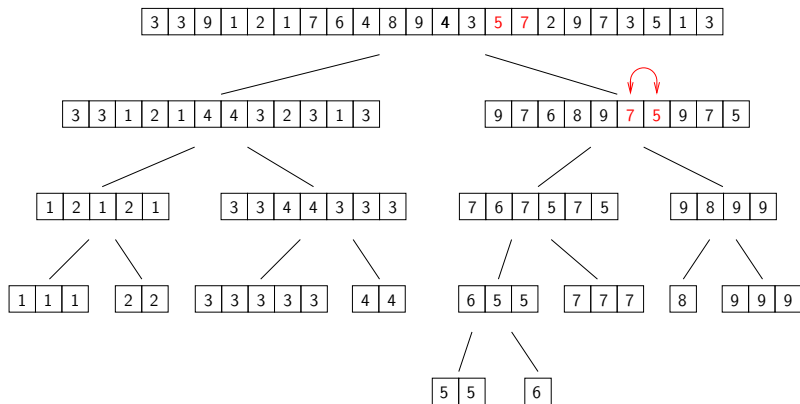
Contiguous Swap



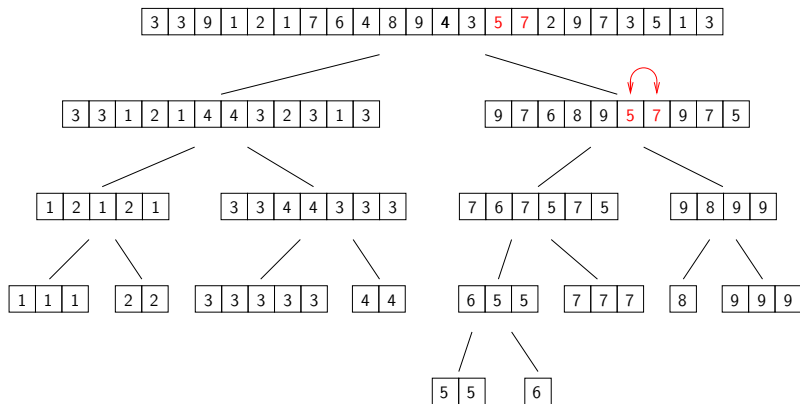
Contiguous Swap



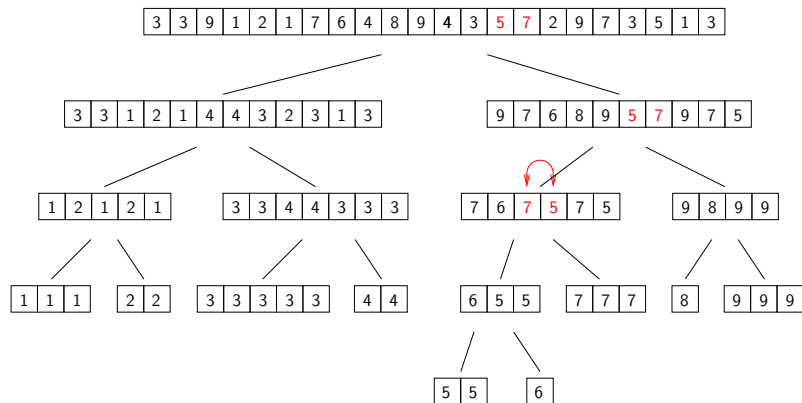
Contiguous Swap



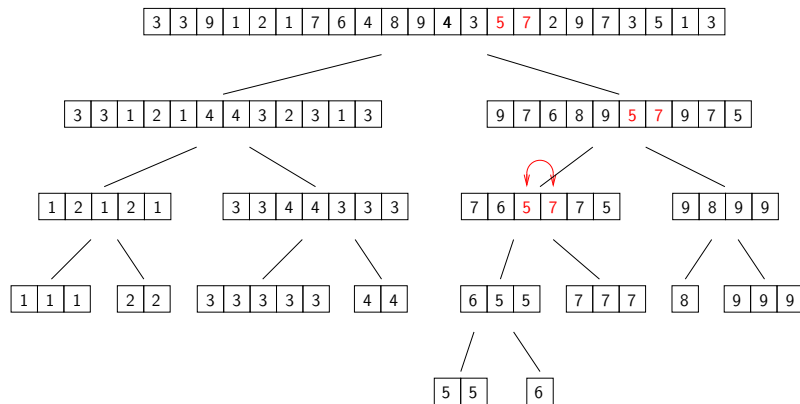
Contiguous Swap



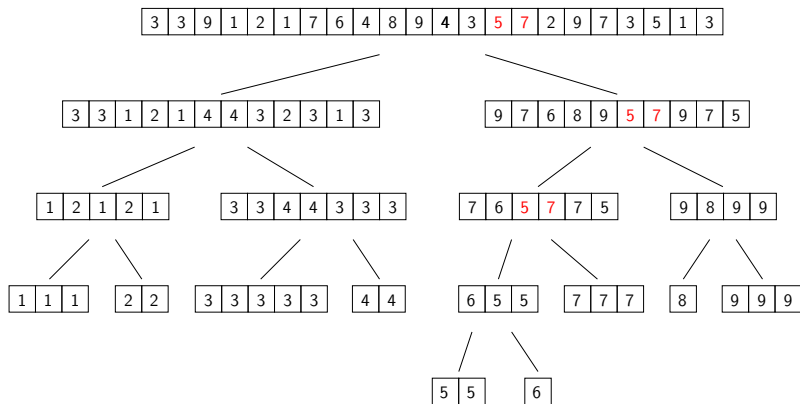
Contiguous Swap



Contiguous Swap



Contiguous Swap



Wavelet Trees support many other queries

- ▶ How many elements with values between a and b ?
- ▶ Rectangular queries in a grid
- ▶ Augment with other structures in every level

Wavelet Trees support many other queries

- ▶ How many elements with values between a and b ?
- ▶ Rectangular queries in a grid
- ▶ Augment with other structures in every level
- ▶ Push/pop back/front

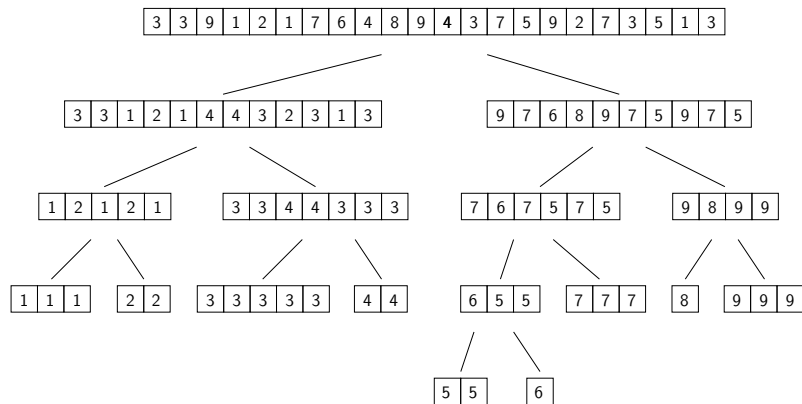
Wavelet Trees support many other queries

- ▶ How many elements with values between a and b ?
- ▶ Rectangular queries in a grid
- ▶ Augment with other structures in every level
- ▶ Push/pop back/front

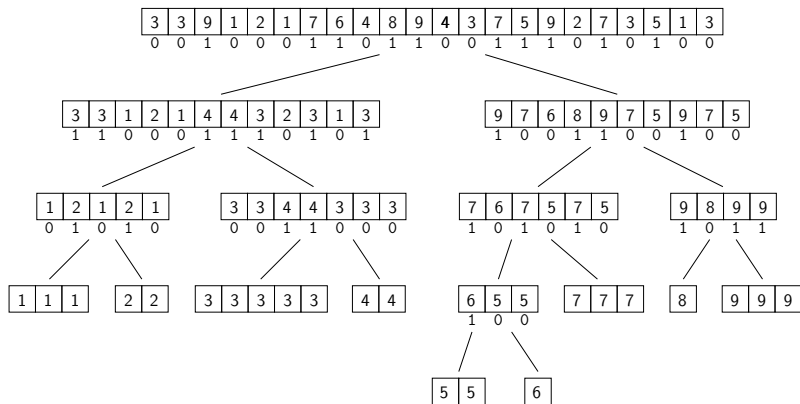
and with a bit more effort

- ▶ Update the value of arbitrary elements in the list
(not in the paper)

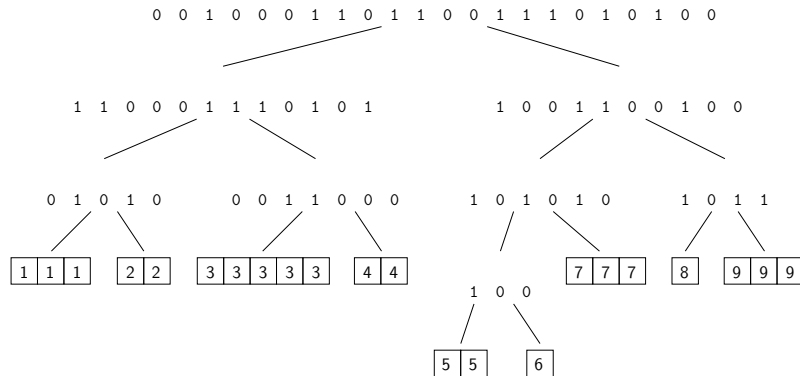
Wavelet Tree implementation



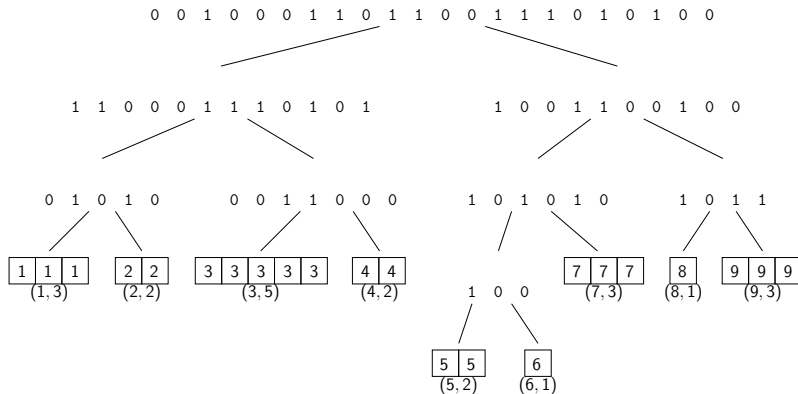
Wavelet Tree implementation



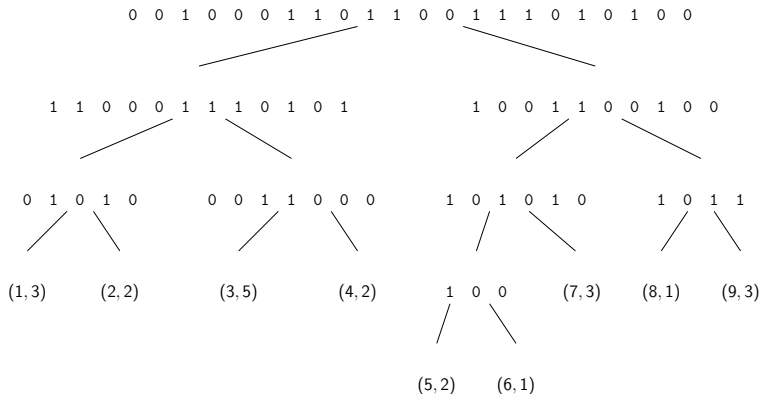
Wavelet Tree implementation



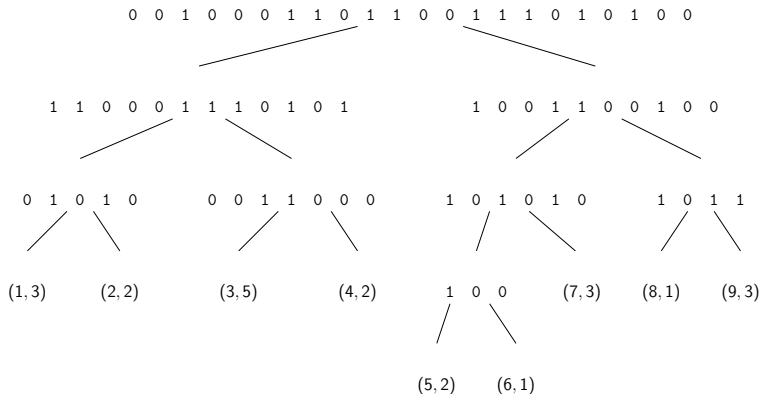
Wavelet Tree implementation



Wavelet Tree implementation

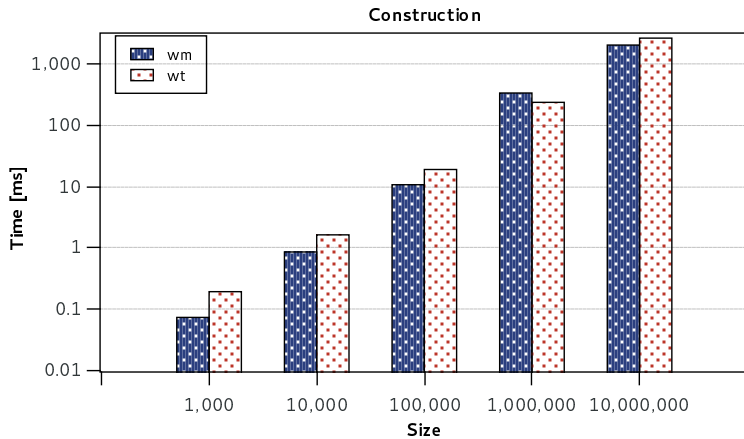


Wavelet Tree implementation

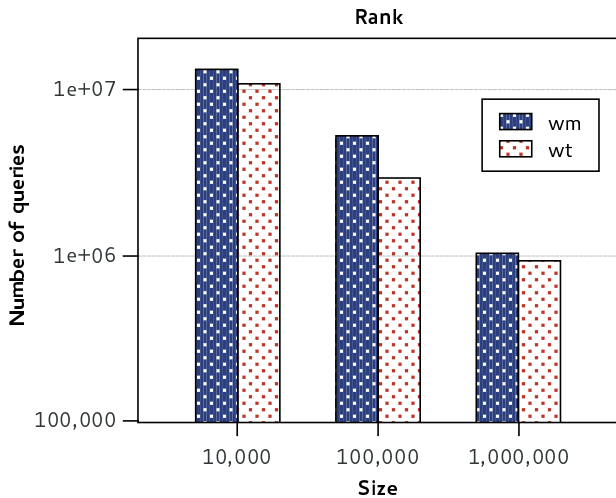


<https://github.com/nilehmann/wavelet-tree>

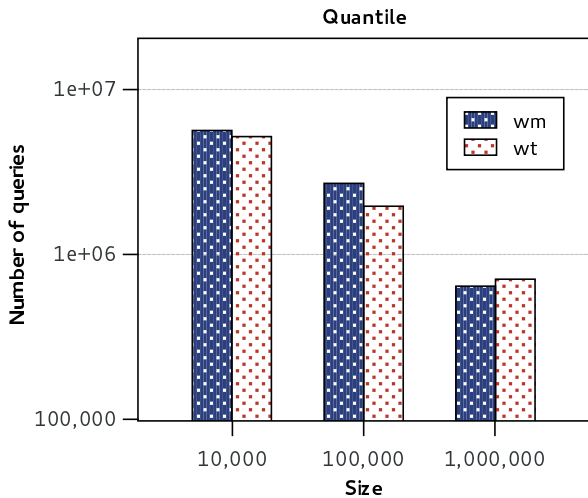
Performance



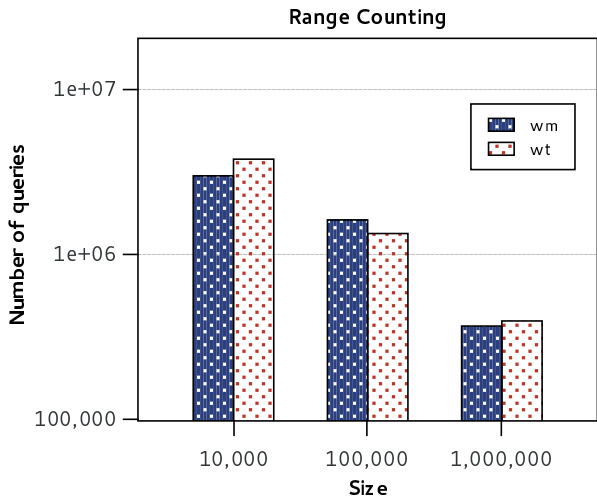
200ms for input of size 10^6
2s for input of size 10^7



10^6 rank queries in 1s for input of size 10^6 .



$\sim 10^6$ quantile queries in 1s for input of size 10^6 .



$\sim 10^5$ range queries in 1s for input of size 10^6 .

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

As of today:

	Submitted	Accepted
ILKQ1	49	9
ILKQ2	32	6
ILKQ3	35	2

Submissions until April 2016

As of today: two users submitted correct solutions using Wavelet Trees!

Wavelet Trees for Competitive Programming

Robinson Castro, Nico Lehmann, Jorge Pérez, Bernardo Subercaseaux

Universidad de Chile, École Centrale Paris

Outline

Motivation

The Wavelet Tree

Queries

Implementation

Performance

Conclusions